Week 11 - Monday

COMP 2230

Last time

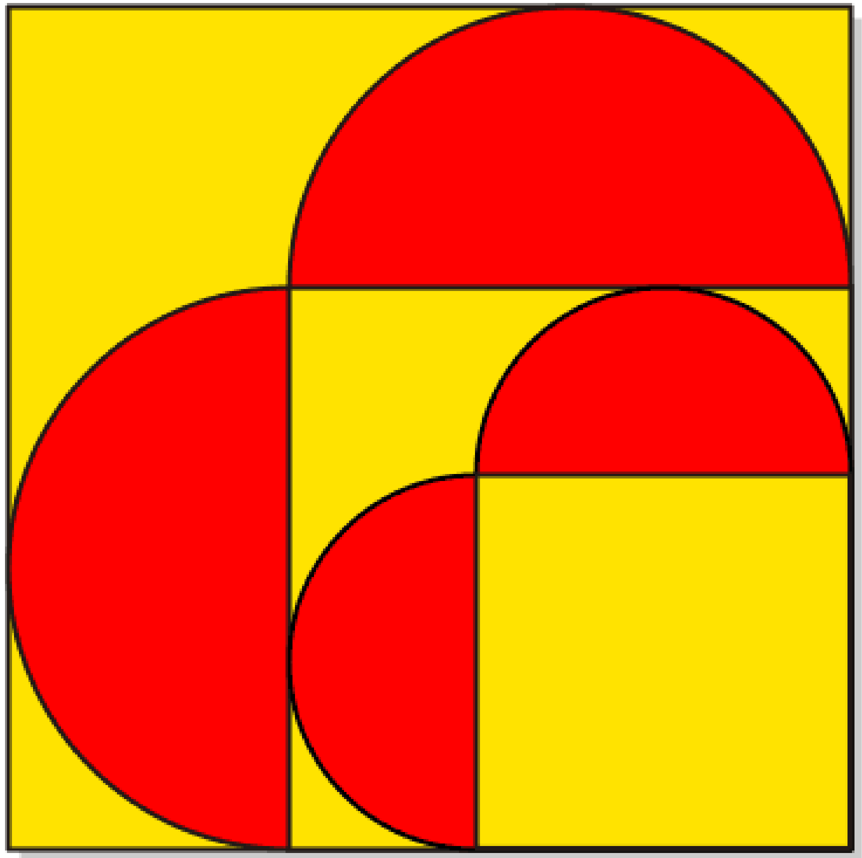
- Trees
- Rooted trees
- Spanning trees

Questions?

Assignment 5

Logical warmup

- Which has a bigger total area?
  - Yellow
  - Red



Spanning Trees

Finding a minimum spanning tree

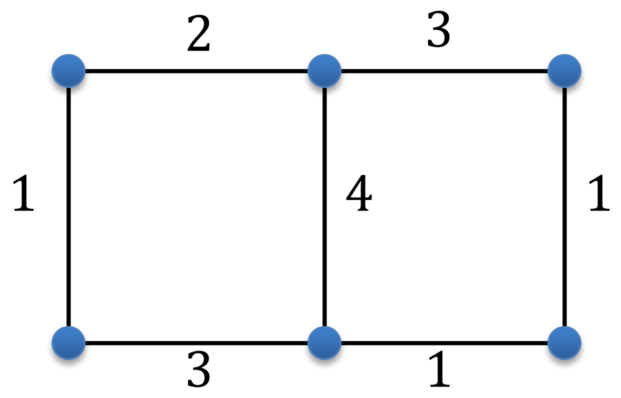
- Kruskal's algorithm gives an easy to follow technique for finding an MST on a weighted, connected graph
- Informally, go through the edges, adding the smallest one, unless it forms a circuit
- **Algorithm:**
  - Input: Graph G with n vertices
  - Create a subgraph T with all the vertices of G (but no edges)
  - Let E be the set of all edges in G
  - Set $m = 0$
  - While $m < n - 1$
    - Find an edge e in E of least weight
    - Delete e from E
    - If adding e to T doesn't make a circuit
      - Add e to T
      - Set $m = m + 1$
  - Output: T

Prim's algorithm

- Prim's algorithm gives another way to find an MST
- Informally, start at a vertex and add the next closest node not already in the MST
- **Algorithm:**
  - Input: Graph G with n vertices
  - Let subgraph T contain a single vertex v from G
  - Let V be the set of all vertices in G except for v
  - For i from 1 to $n - 1$
    - Find an edge e in G such that:
      - e connects T to one of the vertices in V
      - e has the lowest weight of all such edges
    - Let w be the endpoint of e in V
    - Add e and w to T
    - Delete w from V
  - Output: T

Prim fights Kruskal

- Apply Kruskal's algorithm to the graph below
- Now, apply Prim's algorithm to the graph below
- Is there any other MST we could make?



Graphing Functions and Big-O, Big-Omega, and Big-Theta Notations

Three-Sentence Summary

Graphing Functions

Graphing functions

- I am assuming that you've been graphing functions since about 6th or 7th grade
- Still, the formal definition of a graph of a function is:
  - Let f be a real-valued function of a real variable
  - The **graph of** f is the set of all points (x, y) in the Cartesian coordinate plane such that x is in the domain of f and $y = f(x)$

Power functions

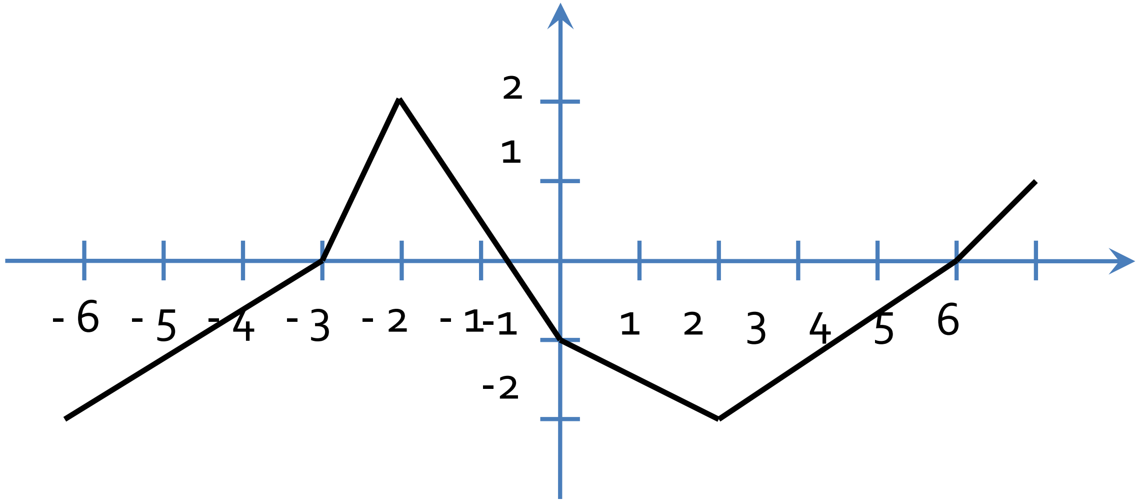
- In computer science, we are often interested in **power functions of** x written $p_a(x)$ where $p_a(x) = x^a$ where x and a are nonnegative
- Power functions are the building blocks of polynomial functions
- Graph the following on the same coordinate axes
  - p_0
  - $p_{\frac{1}{2}}$
  - p_1
  - p_2

Discontinuous functions

- Recall the definition of the floor of x:
  - $\lfloor x \rfloor$ = the largest integer that is less than or equal to x
- Graph $f(x) = \lfloor x \rfloor$
- Defining functions on integers instead of real values affects their graphs a great deal
- Graph $p_1(x) = x, x \in \mathbb{R}$
- Graph $f(n) = n, n \in \mathbb{N}$

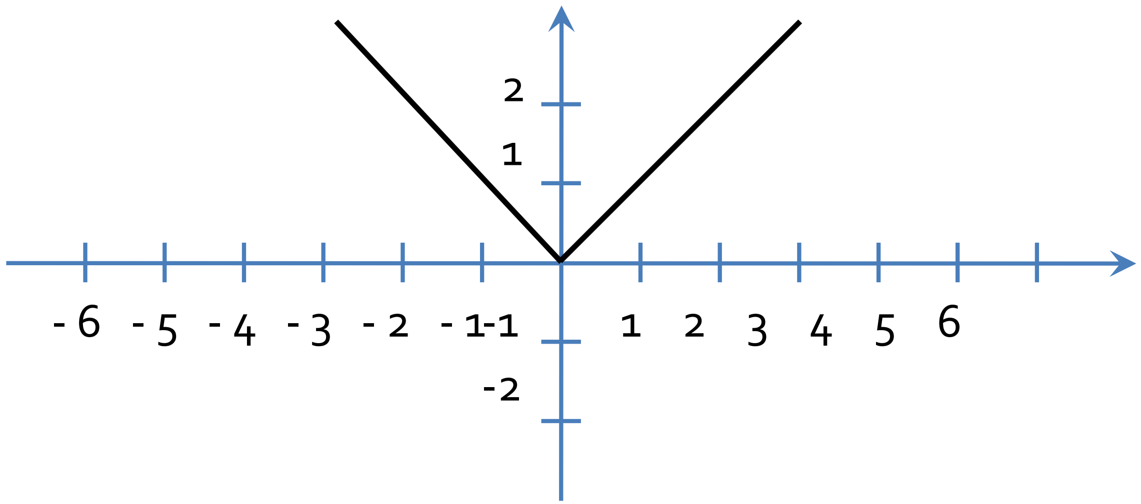
Multiples of functions

- There is a strong visual (and of course mathematical) correlation to a function that is the multiple of another function
- Examples:
  - $g(x) = x + 2$
  - $2 \cdot g(x) = 2x + 4$
- Given f graphed below, sketch $2f$



Absolute value

- Consider the absolute value function
  - $f(x) = |x|$



- Left of the origin it is constantly **decreasing**
- Right of the origin it is constantly **increasing**

Increasing and decreasing functions

- We say that f is **decreasing on the set** S iff for all real numbers x_1 and x_2 in S, if $x_1 < x_2$, then $f(x_1) > f(x_2)$
- We say that f is **increasing on the set** S iff for all real numbers x_1 and x_2 in S, if if $x_1 < x_2$, then $f(x_1) < f(x_2)$
- We say that f is an **increasing** (or **decreasing**) function iff f is increasing (or decreasing) on its entire domain
- A positive multiple of an increasing function is increasing
- Virtually all running time functions are increasing functions

Asymptotic Notations

Growth of functions

- Mathematicians worry about the growth of various functions
- They usually express such things in terms of limits, maybe with derivatives
- We are focused primarily on functions that bound running time spent and memory consumed
- We just need a rough guide
- We want to know the **order** of the growth

Definitions

- Let f and g be real-valued functions defined on the same set of nonnegative real numbers
- f **is of order at least** g, written $f(x)$ is $\Omega(g(x))$, iff there is a positive $A \in \mathbb{R}$ and a nonnegative $a \in \mathbb{R}$ such that
  - $A|g(x)| \leq |f(x)|$ for all $x > a$
- f **is of order at most** g, written $f(x)$ is $O(g(x))$, iff there is a positive $B \in \mathbb{R}$ and a nonnegative $b \in \mathbb{R}$ such that
  - $|f(x)| \leq B|g(x)|$ for all $x > b$
- f **is of order** g, written $f(x)$ is $\Theta(g(x))$, iff there are positive $A, B \in \mathbb{R}$ and a nonnegative $k \in \mathbb{R}$ such that
  - $A|g(x)| \leq |f(x)| \leq B|g(x)|$ for all $x > k$

Using the notation

- Express the following statements using appropriate notation:
  - $10|x^6| \leq |17x^6 - 45x^3 + 2x + 8| \leq 30|x^6|$, for $x > 2$
  - $15|\sqrt{x}| \leq \left|\frac{15\sqrt{x}(2x+9)}{x+1}\right|, x > 0$
  - $\left|\frac{15\sqrt{x}(2x+9)}{x+1}\right| \leq 15|\sqrt{x}|, x > 6$

- Justify the following:
  - $\left|\frac{15\sqrt{x}(2x+9)}{x+1}\right|$ is $\Theta(\sqrt{x})$

Properties of Ω-, O-, and Θ-notation

- Let f, g, h, and k be real-valued functions defined on the same set of nonnegative real numbers

1. $f(x)$ is $\Omega(g(x))$ and $f(x)$ is $O(g(x))$ iff $f(x)$ is $\Theta(g(x))$
2. $f(x)$ is $\Omega(g(x))$ iff $g(x)$ is $O(f(x))$
3. $f(x)$ is $\Omega(f(x))$, $f(x)$ is $O(f(x))$, and $f(x)$ is $\Theta(f(x))$
4. If $f(x)$ is $O(g(x))$ and $g(x)$ is $O(h(x))$ then $f(x)$ is $O(h(x))$
5. If $f(x)$ is $O(g(x))$ and c is a positive real, then $c \cdot f(x)$ is $O(g(x))$
6. If $f(x)$ is $O(h(x))$ and $g(x)$ is $O(k(x))$ then $f(x) + g(x)$ is $O(G(x))$ where $G(x) = \max(|h(x)|, |k(x)|)$ for all x
7. If $f(x)$ is $O(h(x))$ and $g(x)$ is $O(k(x))$ then $f(x) \cdot g(x)$ is $O(h(x) \cdot k(x))$

Ticket Out the Door

Upcoming

Next time…

- Analysis of algorithm efficiency I
- Exponential and logarithmic functions

Reminders

- **Work on Assignment 5**
- Read 11.3 and 11.4
  - Prepare a three-sentence summary
  - Extra credit if you get called on